Multi-Population Adaptive Inflationary Differential Evolution

Marilena Di Carlo, Massimiliano Vasile, Edmondo Minisci

Department of Mechanical and Aerospace Engineering
University of Strathclyde

marilena.di-carlo@strath.ac.uk

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Introduction

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  Adaptive Inflationary Differential Evolution Algorithm, AIDEA
- **Multi-population version of AIDEA (MP-AIDEA)**

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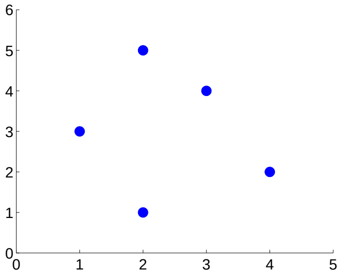
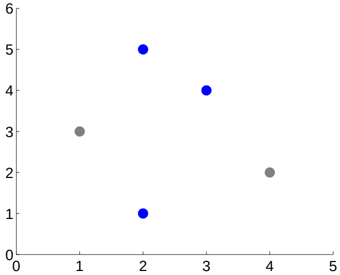
Differential Evolution

- Initialize population in the search space



Differential Evolution



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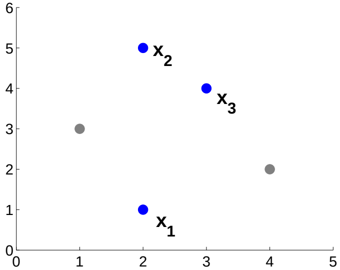

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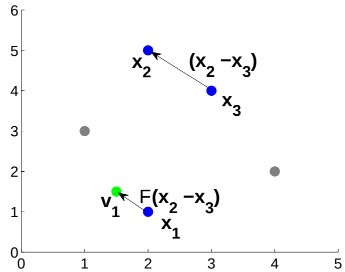

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- Select three individuals $\mathbf{x}_1$, $\mathbf{x}_2$ and $\mathbf{x}_3$
- Apply mutation:
  $\mathbf{v}_1 = \mathbf{x}_1 + F \cdot (\mathbf{x}_2 - \mathbf{x}_3)$

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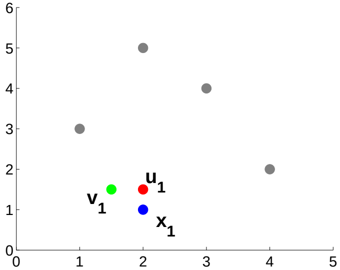

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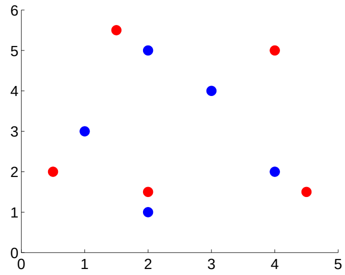
  $\mathbf{v}_1 = \mathbf{x}_1 + F \cdot (\mathbf{x}_2 - \mathbf{x}_3)$
- Apply crossover to obtain trial vector $\mathbf{u}_1$:
  $$u_1^j = \begin{cases} v_1^j, & \text{if } \text{rand}(0,1) \leq CR \text{ or } j = j_{rand} \\ x_1^j, & \text{otherwise} \end{cases}$$

Differential Evolution



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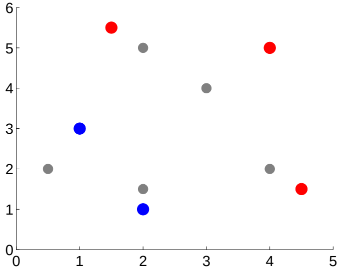
Differential Evolution



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- Repeat operation for all the individuals
- Survival selection:
  $$\mathbf{x}_i' = \begin{cases} \mathbf{u}_i, & \text{if } f(\mathbf{u}_i) \leq f(\mathbf{x}_i) \\ \mathbf{x}_i, & \text{otherwise} \end{cases}$$

IDEA

- DE drawbacks:
  - Stagnation of the optimization process
  - CR and F difficult to tune and heavily problem dependent

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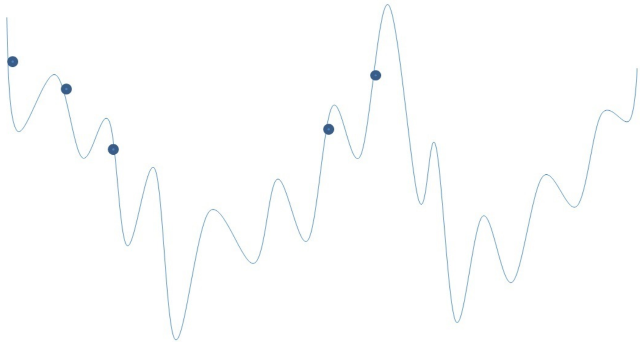
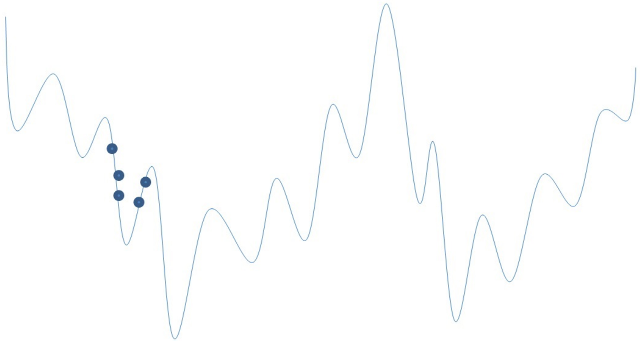
  - CR and F difficult to tune and heavily problem dependent
- IDEA (Inflationary Differential Evolution Algorithm)
  M. Vasile, E. Minisci, M. Locatelli, 2011
  - Hybridization of DE with the restarting procedure of Monotonic Basin Hopping (MBH) algorithm

IDEA



1. Initialize population in the search space and run Differential Evolution

IDEA



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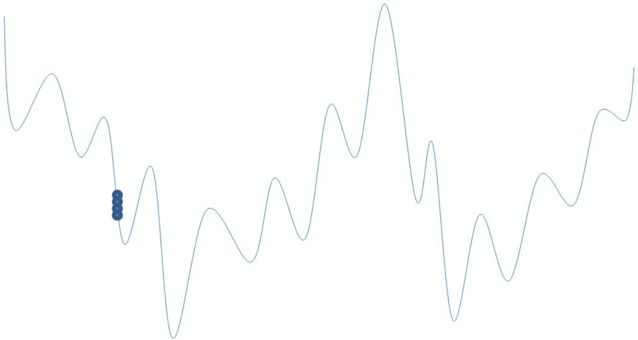
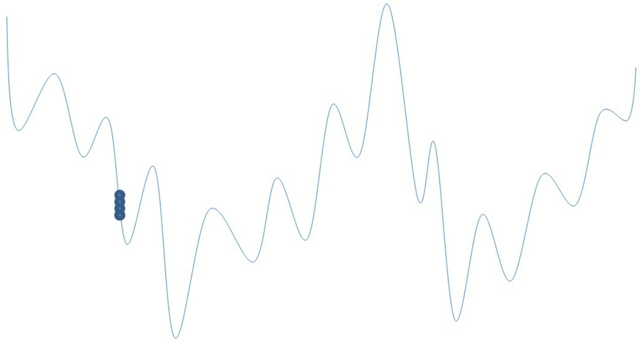


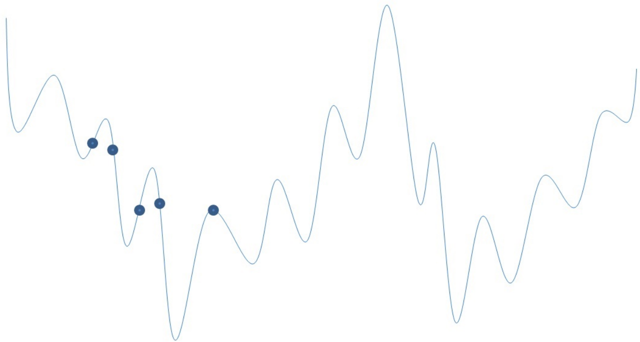
2. Population contraction: $r \leq \rho \cdot r_{max}$
   $r = \max\left(\|\mathbf{x}_i - \mathbf{x}_j\|\right)$ and r_{max} is the maximum value of r recorded during the convergence

IDEA



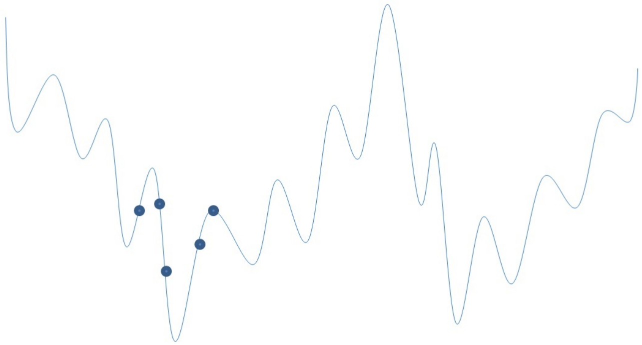
3. Perform **local search** from the best individual in the population and locate local minimum

IDEA



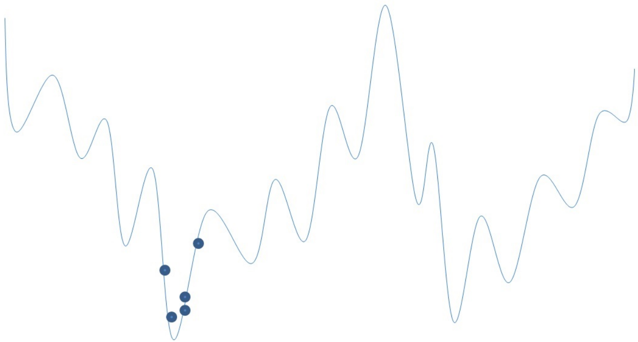
4. **Restart population in a bubble** of dimension δ_{local} around the local minimum

IDEA



5. Repeat DE until convergence

IDEA



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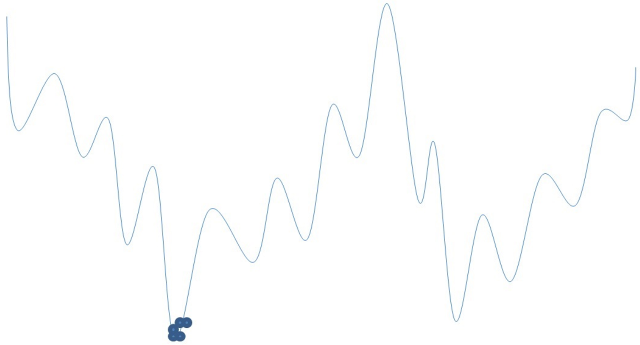

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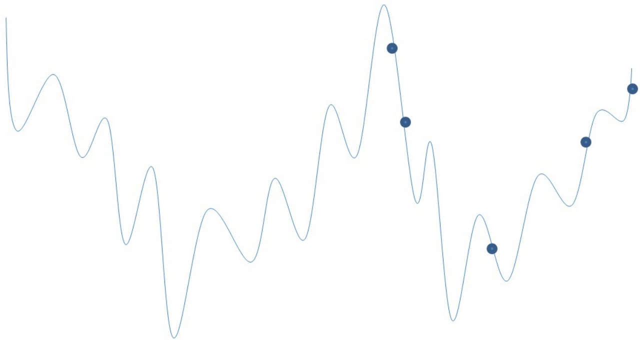

6. When more than n_{LR} local restarts have been performed **globally restart the population** at a distance δ_{global} from the centers of the clusters of local minima

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- AIDEA (Adaptive Inflationary Differential Evolution Algorithm)
  E. Minisci, M. Vasile, 2014
  - Hybridization of DE with the restaring procedure of Monotonic Basin Hopping (MBH) algorithm
  - Adaptation of the DE parameters *CR* and *F*

AIDEA

CR and F adaptation

- Initialization of CRF, regular mesh in the space:

  $CR \in [0.1, 0.99]$
  $F \in [-0.5, 1]$

AIDEA

CR and *F* adaptation

- Initialization of *CRF*, regular mesh in the space:
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- Sample of *CR* and *F* values from the Parzen distribution associated to *CRF* and association of each (CR, F) couple to an individual of the population

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$$d = f(\mathbf{x}') - f(\mathbf{x})$$

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- *CRF* update: (CR, F) couples with lower d are substituted by (CR, F) couples with higher d

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- Strategies for the generation of the mutant vector:
  - DE/best-DE/rand
  - DE/arch-DE/rand
  - DE/arch-DE/best

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- Strategies for the generation of the mutant vector:
  - DE/best-DE/rand
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  - DE/arch-DE/best
- *CR* and *F* adaptation:
  - MP-AIDEA-CRF1
    Same *CR* and *F* values for every individual of the same population
  - MP-AIDEA-CRF2
    Different *CR* and *F* values for each element of each population

MP-AIDEA

δ_{local} adaptation

- Computation of the minimum and maximum distance between all local minima, d_{minMIN} and d_{minMAX}

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- Computation for each population of the distance between consecutive local minima:
  $p = \left\| \mathbf{x}_{min}^{k+1} - \mathbf{x}_{min}^{k} \right\|$

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- Computation for each population of the distance between consecutive local minima: $p = \left\| \mathbf{x}_{min}^{k+1} - \mathbf{x}_{min}^{k} \right\|$
- Update of B: population with higher values of p are characterized by a better value of δ_{local}

MP-AIDEA versions

	CRF1	CRF2	DE-mut1	DE-mut2	DE-mut3	δ_{local}
MP-AIDEA 1	✓		✓			
MP-AIDEA 2	✓			✓		
MP-AIDEA 2	✓				✓	
MP-AIDEA 4	✓		✓			✓
MP-AIDEA 5	✓			✓		✓
MP-AIDEA 6	✓				✓	✓
MP-AIDEA 7		✓	✓			
MP-AIDEA 8		✓		✓		
MP-AIDEA 9		✓			✓	
MP-AIDEA 10		✓	✓			✓
MP-AIDEA 11		✓		✓		✓
MP-AIDEA 12		✓			✓	✓

DE-mut1: DE/best-DE/rand

DE-mut2: DE/arch-DE/rand

DE-mut3: DE/arch-DE/best

MP-AIDEA versions

	CRF1	CRF2	DE-mut1	DE-mut2	DE-mut3	δ_{local}
MP-AIDEA 1	✓		✓			
MP-AIDEA 2	✓			✓		
MP-AIDEA 2	✓				✓	
MP-AIDEA 4	✓		✓			✓
MP-AIDEA 5	✓			✓		✓
MP-AIDEA 6	✓				✓	✓
MP-AIDEA 7		✓	✓			
MP-AIDEA 8		✓		✓		
MP-AIDEA 9		✓			✓	
MP-AIDEA 10		✓	✓			✓
MP-AIDEA 11		✓		✓		✓
MP-AIDEA 12		✓			✓	✓

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DE-mut2: DE/arch-DE/rand

DE-mut3: DE/arch-DE/best

Test Cases

Competition of the Congress on Evolutionary Computation (CEC)

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1. Spread Spectrum Radar Polyphase Code Design, CEC 2011
2. Tersoff Radar Function Minimization Problem, CEC 2011
3. Schwefel's Problem, CEC 2005
4. Rotated Version of Hybrid Composition Function, CEC 2005

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Algorithm performance:

- Success rate: number of successful runs over 100 total runs
- Successful run: $f(\mathbf{x}_{min}) < f_{min} + \epsilon$

  CEC 2011: $\epsilon = 0.001$

  CEC 2005: $\epsilon = 0.01$

Spread Spectrum Radar Polyphase Code Design

Spread Spectrum Radar Polyphase Code Design

- Problem and algorithm parameters

D	f_{min}	FEs	δ_{local}*	δ_{global}	ρ	n_{LR}
20	0.5	$1.5 \cdot 10^5$	0.1	0.1	0.2	10

* for AIDEA and MP-AIDEA versions which do not adapt δ_{local}

Spread Spectrum Radar Polyphase Code Design

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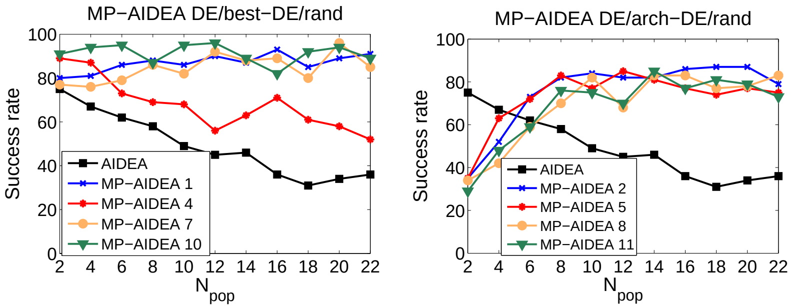
- Algorithm comparison
  - AIDEA
  - Best performing algorithms of CEC 2011 competition:

    GA-MPC (Genetic Algorithm with Multi-Parent Crossover)
    WI-DE (Weed Inspired Differential Evolution)

Spread Spectrum Radar Polyphase Code Design

- MP-AIDEA and AIDEA success rate



Spread Spectrum Radar Polyphase Code Design

- Statistics of the results

Algorithm	Min	Mean	Max	Str.Dev.
MP-AIDEA 10	0.5000	0.5045	0.5690	0.0135
AIDEA	0.5000	0.5130	0.6422	0.0263
GA-MPC	0.5000	0.7484	0.9334	0.1249
WI-DE	0.5000	0.6560	0.9931	0.1160

Tersoff Potential Function Minimization Problem

Tersoff Potential Function Minimization Problem

- Problem and algorithm parameters

D	f_{min}	FEs
30	-36.9286	$1.5 \cdot 10^5$

	δ_{local}*	δ_{global}	ρ	n_{LR}
Case 1	**0.1**	0.1	0.2	10
Case 2	**0.3**	0.1	0.2	10

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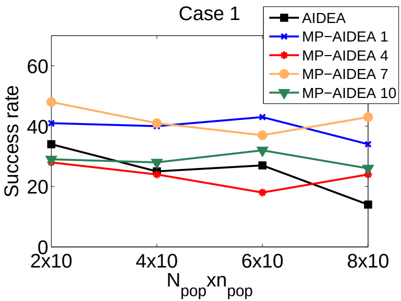
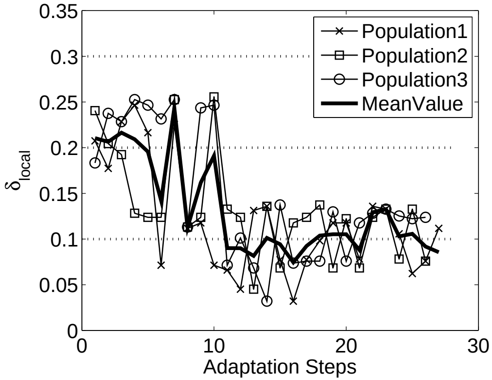


Tersoff Potential Function Minimization Problem

- δ_{local} adaptation

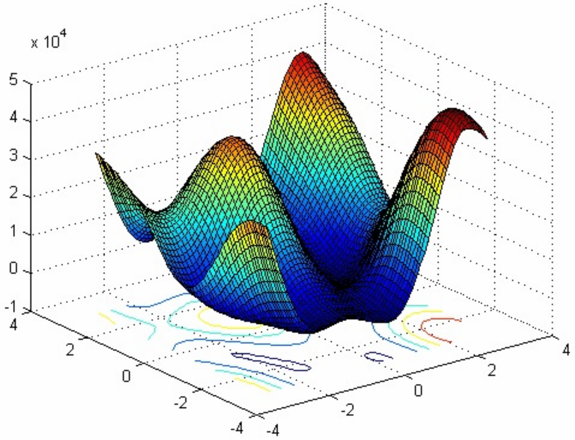


Tersoff Potential Function Minimization Problem

- Statistics of the results

Algorithm	Min	Mean	Max	Str.Dev.
Case 1				
MP-AIDEA 7	-36.9286	-36.7120	-34.3504	0.3835
AIDEA	-36.9286	-36.8046	-35.9700	0.2483
Case 2				
MP-AIDEA 10	-36.9286	-36.6689	-34.1647	0.4399
AIDEA	-36.9286	-36.6219	-35.4467	0.4694
GA-MPC	-36.8457	-35.03883	-34.1076	0.8329
WI-DE	-36.8	-35.6	-34.2	0.904

Schwefel's Problem



Schwefel's Problem

- Problem and algorithm parameters

D	f_{min}	FEs	δ_{local}*	δ_{global}	ρ	n_{LR}
30/50	-460	$3 \cdot 10^5$ / $5 \cdot 10^5$	0.1	0.1	0.2	5

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Schwefel's Problem

- Problem and algorithm parameters

D	f_{min}	FEs	δ_{local}*	δ_{global}	ρ	n_{LR}
30/50	-460	$3{\cdot}10^5$ / $5{\cdot}10^5$	0.1	0.1	0.2	5

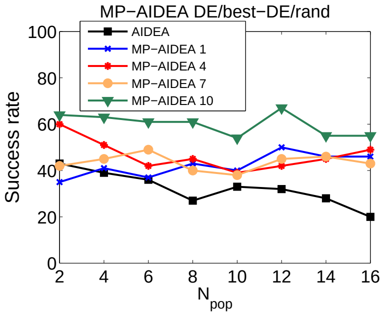
* for AIDEA and MP-AIDEA versions which do not adapt δ_{local}

- Algorithm comparison
  - AIDEA
  - Best performing algorithms of CEC 2005 competition:

    IPOP-CMA-ES (Increasing Population Size Covariance Matrix Adaptation Evolution Strategy)

Schwefel's Problem

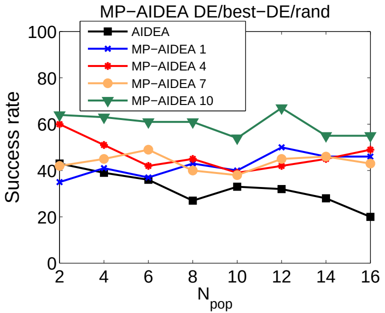
- MP-AIDEA and AIDEA success rate



$$D = 30$$
$$n_{pop} = 10$$

Schwefel's Problem

- MP-AIDEA and AIDEA success rate



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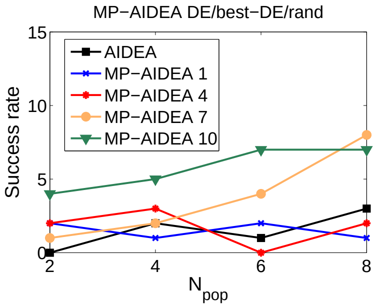


$D = 50$
$n_{pop} = 20$

Schwefel's Problem

- Statistics of the results (error values w.r.t. f_{min})

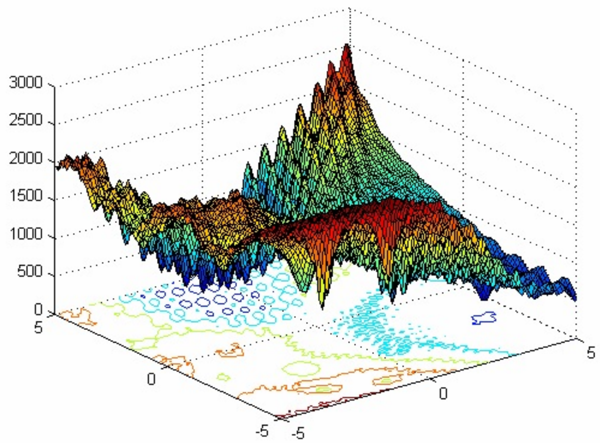
Algorithm	Min	Mean	Max	Str.Dev.
D = 30				
MP-AIDEA 10	1.39e-9	2.45e+1	4.77e+2	7.25e+1
AIDEA	2.01e-9	1.03e+2	1.00e+3	1.97e+2
IPOP-CMA-ES	3.79e-9	4.43e+4	1.10e+6	2.19e+5
D = 50				
MP-AIDEA 10	2.48e-8	8.91e+2	9.54e+3	1.27e+3
AIDEA	5.61e-8	2.22e+3	1.33e+4	2.69e+3
IPOP-CMA-ES	9.67e+0	2.27e+5	5.57e+6	1.11e+6

Schwefel's Problem

- Statistics of the results (error values w.r.t. f_{min})

Algorithm	Min	Mean	Max	Str.Dev.
D = 30				
MP-AIDEA 10	1.39e-9	2.45e+1	4.77e+2	7.25e+1
AIDEA	2.01e-9	1.03e+2	1.00e+3	1.97e+2
IPOP-CMA-ES	3.79e-9	4.43e+4	1.10e+6	2.19e+5
D = 50				
MP-AIDEA 10	**2.48e-8**	8.91e+2	9.54e+3	1.27e+3
AIDEA	**5.61e-8**	2.22e+3	1.33e+4	2.69e+3
IPOP-CMA-ES	**9.67e+0**	2.27e+5	5.57e+6	1.11e+6

Rotated Version of Hybrid Composition Function



Rotated Version of Hybrid Composition Function

- Problem and algorithm parameters

D	f_{min}	FEs	δ_{local}*	δ_{global}	ρ	n_{LR}
10	120	$1 \cdot 10^5$	0.1	0.1	0.2	5

* for AIDEA and MP-AIDEA versions which do not adapt δ_{local}

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- Algorithm comparison
  - AIDEA
  - Best performing algorithms of CEC 2005 competition:

    IPOP-CMA-ES (Increasing Population Size Covariance Matrix Adaptation Evolution Strategy)

Rotated Version of Hybrid Composition Function

- MP-AIDEA success rate

Algorithm	2x20	4x20	6x20	8x20
MP-AIDEA 4	**2**	**1**	**1**	0
MP-AIDEA 10	**1**	0	**1**	**3**

Rotated Version of Hybrid Composition Function

- MP-AIDEA success rate

Algorithm	2x20	4x20	6x20	8x20
MP-AIDEA 4	**2**	**1**	**1**	0
MP-AIDEA 10	**1**	0	**1**	**3**

- Statistics of the results (error values w.r.t. f_{min})

Algorithm	Min	Mean	Max	Str.Dev.
MP-AIDEA 10	**7.44e-11**	1.05e+2	1.32e+2	2.35e+1
AIDEA	**5.38e+1**	1.02e+2	1.14e+2	8.42e+0
IPOP-CMA-ES	**7.92e+1**	9.13e+1	9.68e+1	3.49e+0

Conclusions

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Conclusions

- Multi-population version of AIDEA
  - Hybridization of DE and MBH
  - Adaptation of CR, F and δ_{local}
- Test results:
  - MP-AIDEA with adaptation of δ_{local} give good results and do not require the setting of this parameter
  - **Most successful versions of MP-AIDEA were able to locate for the first time the global minima of two difficult academic test functions**

Future work

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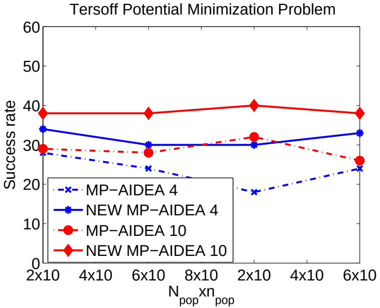
- Adaptation of other parameters:
  maximum number of local restart n_{LR}



Thank you for your attention