

A PARTICLE SWARM OPTIMIZATION HYPER-HEURISTIC FOR THE DYNAMIC VEHICLE ROUTING PROBLEM

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Abstract This paper presents a method for choosing a Particle Swarm Optimization based optimizer for the Dynamic Vehicle Routing Problem on the basis of the initially available data of a given problem instance. The optimization algorithm is chosen on the basis of a prediction made by a linear model trained on that data and the relative results obtained by the optimization algorithms. The achieved results suggest that such a model can be used in a hyper-heuristic approach as it improved the average results, obtained on the set of benchmark instances, by choosing the appropriate algorithm in 82% of significant cases. Two leading multi-swarm Particle Swarm Optimization based algorithms for solving the Dynamic Vehicle Routing Problem are used as the basic optimization algorithms: Khouadjia's et al. Multi-Environmental Multi-Swarm Optimizer and authors' 2-Phase Multiswarm Particle Swarm Optimization.

Keywords: Dynamic vehicle routing problem, Hyper-heuristic, Particle swarm optimization.

1. Introduction

Dynamic transportation problems have been considered in the literature by Psaraftis [20, 21] since 1980. After the introduction of a set benchmarks by Kilby [14] and Montemanni [15], several meta-heuristic based algorithms have been developed in order to solve the problem, including Ant Colony Optimization (ACS) [6, 15], Genetic Algorithm (GA) [9, 7], Tabu Search (TS) [9], and Particle Swarm Optimization (PSO) [13, 17].

Although some of the works [7, 9, 12, 16] mention the features of a spatial distribution of the requests of a given DVRP instance (describing it

as spatially uniform or clustered), none of those methods directly use the information about the known requests location and volume. An initial non-parametric approach for using the information about the available requests volumes in order to generate artificial requests (to account for the unknown ones) has been presented by the authors [18].

This paper proposes a hyper-heuristic method based on Multi-Environmental Multi-Swarm Optimizer (MEMSO) [11, 12, 13] and 2-Phase Multiswarm Particle Swarm Optimization (2MPSO) [16, 17, 18] algorithms. The hyper-heuristic uses the statistical data about the initially known set of requests in the given Dynamic Vehicle Routing Problem (DVRP).

The rest of the paper is organized as follows. Section 2 defines the DVRP solved in this paper. Section 3 introduces PSO and algorithms MEMSO and 2MPSO (both based on the PSO) used for optimizing DVRP. Section 4 presents the hyper-heuristic approach for solving the DVRP. Section 5 gives experimental setup and results obtained by the method. Finally, Section 6 concludes the paper.

2. Dynamic Vehicle Routing Problem

DVRP is a dynamic version of the generalization of a Traveling Salesman Problem (TSP), called Vehicle Routing Problem (VRP). In the VRP the goal is to optimize a total route for a fleet of vehicles with a limited capacity. VRP has been introduced as a *Truck dispatching problem* in 1959 by Dantzig and Ramser [5]. After the technological advancement of the vehicle tracking devices and development of the Geographical Information Systems a notion of a DVRP has been reintroduced by Psaraftis [21]. The problem has attracted more attention after a set of static VRP benchmarks of Christofides [2], Fisher [8] and Tailard [23] has been customized for the DVRP by Kilby et al. in 1998 [14] and Montemanni et al. in 2005 [15].

In this paper a most common variant of the DVRP is solved [19], sometimes referred to as a VRP with Dynamic Requests (VRPwDR) [12]. In this variant a homogeneous fleet of vehicles (identical capacity $c \in \mathbb{R}$ and speed $sp \in \mathbb{R}$) is considered. There is also an additional constraint, that the vehicle may operate only during a *working day* defined by the opening hours of its depot. During that working day a fleet of m vehicles must serve (visit) a set of n requests. Each request is defined by a location $l_i \in \mathbb{R}^2$, an amount of cargo s_i ($0 \leq s_i \leq c$) to be delivered and an amount of time $u_i \in \mathbb{R}$ it takes to provide a service at that location (unload that cargo). The dynamic nature of that optimization problem comes from the fact that new request may arrive during the working day.

The period of time during which new requests are accepted is limited by a *cut-off time* parameter T_{co} . In all the benchmarks considered in this paper there is one depot for all the vehicles and T_{co} is equal to a half of the working day.

In the mathematical sense, the DVRP is a problem of finding a set of location permutations resulting in a shortest total path length under the time and capacity constraints for each of the permutations in which all the locations are visited exactly once. The exact mathematical formulation of the problem can be found in [7, 17, 18].

3. MEMSO and 2MPSO Algorithms

In this section two most successful approaches to solving DVRP: Khouadjia's et al. MEMSO and authors' 2MPSO are presented. Those two methods are the base algorithms for the hyper-heuristic approach proposed in this paper.

Both, the MEMSO and the 2MPSO, use PSO as their base meta-heuristic and 2-OPT [4] as a route optimization heuristic. In both methods the working day is divided into discrete number of time slices, with the instance of the DVRP problem "frozen" within each time slice. Therefore, each method solves a series of dependent static VRP instances during the optimization process. The difference between PSO applications, encoding of the problem and knowledge transfer in the MEMSO and the 2MPSO methods, together with a brief description of the PSO and 2-OPT algorithms, are presented and discussed in this section.

3.1 Particle Swarm Optimization

PSO algorithm is an iterative population based continuous optimization meta-heuristic approach utilizing the concept of Swarm Intelligence. The algorithm has been introduced by Kennedy and Eberhart in 1995 [10] and has been further developed and studied by other researchers [22, 24, 3].

During the optimization process PSO maintains a set of fitness function solutions (called particles). Each particle has its own location $x \in \mathbb{R}^n$ (an n -dimensional fitness function solution proposal), velocity v (a solution change vector), a set of neighbors N (particles which solutions it can observe), a memory of the best observed solution $x_N^{(BEST)}$ and a memory of the best visited solution $x_i^{(BEST)}$.

In each iteration t the location vector x and velocity vector v of i th particle are changed in the following way:

$$x_{i,t} = x_{i,t-1} + v_{i,t-1} \quad (1)$$

$$v_{i,t} = \omega v_{i,t-1} + u_1 c_1 (x_i^{(BEST)} - x_{i,t}) + u_2 c_2 (x_N^{(BEST)} - x_{i,t}) \quad (2)$$

Where ω denotes an inertia factor, c_1 and c_2 are personal and global attraction factors, u_1 and u_2 follow the uniform n -dimensional distribution on $[0, 1]^n$.

3.2 2-OPT Algorithm

2-OPT has been introduced as a heuristic algorithm for solving the TSP in 1958 [4]. Its most distinctive feature is the ability to remove the entanglement of routes. The algorithm operates by iterating over all the pairs of edges of a given route and checking the possibility of optimizing the length of route by swapping the ends of those edges. An example of a single step of the algorithm on a sample directed cycle is presented in Fig. 1.

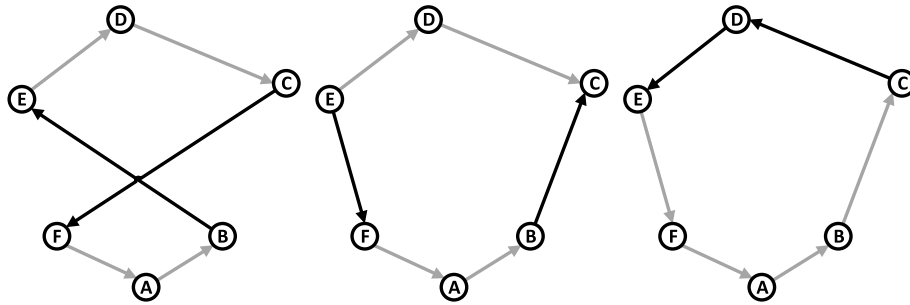


Figure 1: Depiction of a 2-OPT algorithm optimization process over a sample directed cycle. While considering edges BE and CF the algorithm observed that changing them to BC and EF results in a shorter route. After swapping the ends of the considered edges a final step of reversing the direction of the E-D-C path is performed.

2-OPT may be used directly in the VRP variants for optimizing the length of route of a single vehicle.

3.3 Multi-Environmental Multi-Swarm Optimizer

MEMSO [13] algorithm uses PSO to optimize division of the requests among the vehicles. The vehicles' routes within those divided sets are created by a greedy insertion and optimized by the 2-OPT algorithm. The fitness function value is the total length of those routes.

MEMSO uses a discrete encoding of the requests division. The solution is an integer vector representing the requests and the values in the vector are the vehicles' identifiers. Therefore, the PSO algorithm is changed in such a way that a velocity is a vector in $\{1, 2, \dots, m\}^n$ and

addition in eq. (1) is performed in \mathbb{Z}_m^n space (where m is a number of vehicles and n a number of requests).

The knowledge about the current solution is transferred between subsequent time slices by adapting the whole population. For each of the particles the already served and decisively assigned requests are blocked from being changed and new requests are inserted in a greedy way into the solution vectors.

3.4 2-Phase Multi-Swarm Particle Swarm Optimization

2MPSO [17] algorithm uses separate PSO instances to optimize both the division of the requests and their order (in two subsequent optimization phases, hence the name of the method). In the division optimization phase an approach similar to the MEMSO's is used in order to evaluate the total length of routes achieved from the optimized division. The routes are optimized with 2-OPT algorithm from the initially random ordering. In the route optimization phase each of the vehicles is optimized separately and the length of route of the given vehicle is used as a fitness function value.

Instead of customizing the PSO for a discrete problem, 2MPSO follows a continuous optimization approach to applying the PSO algorithm, in contrast with MEMSO. The division of requests among the vehicles is solved as a clustering task, with a number of clusters per vehicle k being the parameter of the method. Therefore, the PSO particle for the first phase optimizer is a sequence of requests clusters centers flattened into a vector in $\mathbb{R}^{2k\hat{m}}$ space (where \hat{m} is the estimated number of vehicles necessary to serve the requests). Particle in the PSO instance optimizing the route of the vehicle is a sequence of requests ranks. Therefore, it is a vector in \mathbb{R}^{n_i} (where n_i is the number of requests assigned to i th vehicle).

The knowledge is transferred between subsequent time slices in a form of cluster centers vector generating a division of requests and a set of requests rank vectors for the routes imposed by requests division. The transferred solution is expanded by a new random cluster center if more vehicles seem to be necessary and the initial ranks of the new requests are also initialized at random.

4. Hyper-Heuristic Approach

Although 2MPSO outperforms MEMSO by 2.22% on average on the Kilby's [14] and Montemanni's [15] sets of benchmarks (see Table 2),

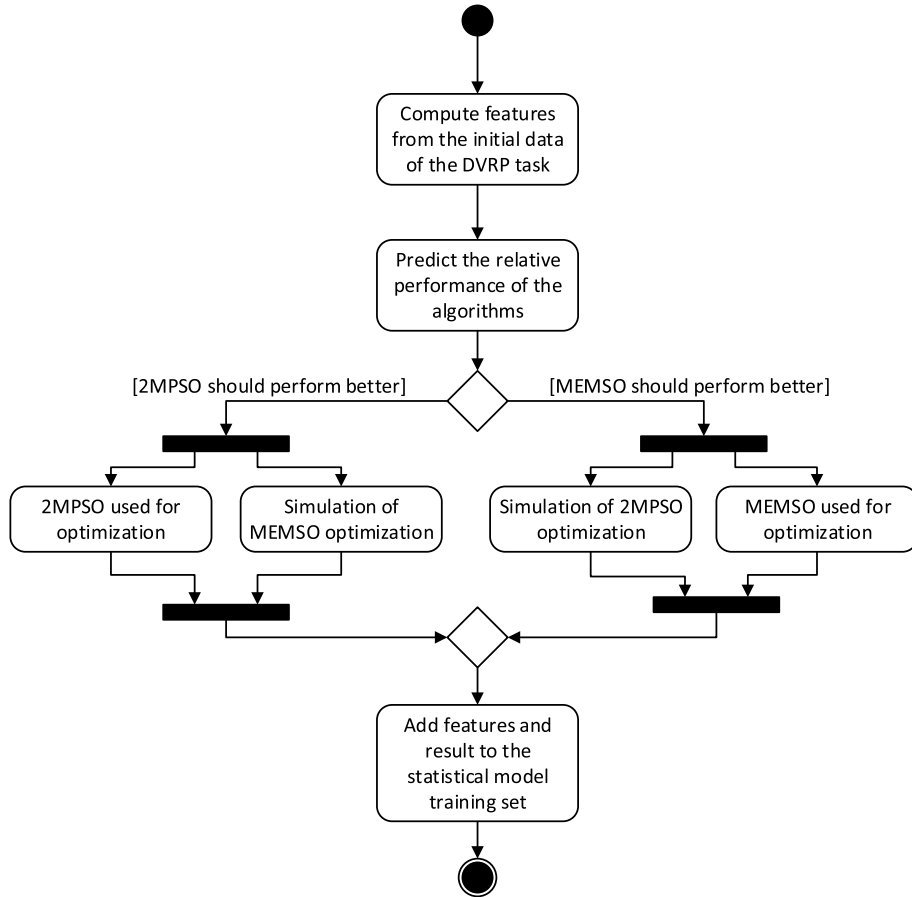


Figure 2: The activity diagram presenting a single run of a 2MPSO and MEMSO based hyper-heuristic.

it might be limited by its clustering approach (or needs an excessively large k) for some particular DVRP tasks.

For that reason, the authors propose a hyper-heuristic approach [1] in which a statistical model might be incrementally trained for choosing the algorithm, which seems to be most suitable for a given DVRP instance. A single run of such hyper-heuristic (solving a single DVRP task) is depicted in Fig. 2. Please observe in the activity diagram, that only the chosen algorithm is used to provide an actual output to some external decision support system. The other algorithm is run only to gather the data and its result is used to tune the prediction model. Please also note, that the choice of the optimization algorithm is done only at the beginning of the optimization process. The prediction model is trained

on the results gathered through a subsequent runs on different DVRP tasks.

The economic cost of running such a hyper-heuristic, in comparison with a single algorithm optimization, would be slightly more than doubled. The doubling comes from the fact, that it is necessary to make a similar amount of computations by two algorithms in order to get comparable results for the performance prediction model. The additional overhead is a result of computations needed for getting statistics from the set of requests and creating a prediction model over the already computed cases. Although the economic cost of proposed approach would definitely be larger than that of a single algorithm, the overhead for the computations necessary for getting solutions during daytime operations would be negligible. The additional operations during the daytime would consist of computing the statistics from the initial state of requests set and providing them to the prediction model. The run of the second algorithm and the training of the prediction model might be done during the nighttime.

This section presents the statistics computed from the benchmark problems and authors' approach to creating a linear model, based on those statistics, predicting the relative MEMSO and 2MPSO algorithms performance.

4.1 Benchmark Characteristics

In order to create a set of features, allowing to discriminate between different benchmark problems, the authors have computed a set of statistics for each of the benchmarks. Those statistics may be divided into 2 groups:

- requests set spatial features,
- requests set volume features.

Within those groups, the following statistics have been computed:

- Spatial features:
 - μ_x, μ_y - mean locations along coordinate system axes,
 - sd_x, sd_y - standard deviation of locations along coordinate system axes,
 - $skew_x, skew_y$ - standardized skewness of locations along coordinate system axes,
 - k_{gap} - gap statistic (estimated optimal number of clusters) for requests locations,

Table 1: Values of the input features computed for the initially known sets of requests from Kilby’s and Montemanni’s benchmarks

Name	μ_x	sd_x	$skew_x$	μ_y	sd_y	$skew_y$	μ_s	sd_s	$skew_s$	nc
c50	0.53	0.32	-0.18	0.51	0.33	0.08	0.09	0.04	0.58	2.00
c75	0.51	0.30	-0.08	0.41	0.29	0.27	0.12	0.06	0.34	4.00
c100	0.48	0.27	0.35	0.46	0.27	0.53	0.08	0.05	0.71	1.00
c100b	0.68	0.17	0.48	0.56	0.26	0.31	0.09	0.05	1.39	0.00
c120	0.38	0.32	0.84	0.67	0.24	0.20	0.06	0.03	1.46	0.67
c150	0.49	0.27	0.06	0.45	0.24	0.28	0.08	0.04	0.78	6.00
c199	0.52	0.26	-0.04	0.47	0.24	0.19	0.09	0.04	0.52	9.00
f71	0.44	0.23	-0.02	0.72	0.18	-0.17	0.04	0.05	1.87	0.00
f134	0.61	0.29	-0.47	0.42	0.22	0.70	0.07	0.11	2.35	1.50
tai75a	0.27	0.18	1.39	0.47	0.20	1.22	0.11	0.16	2.27	3.00
tai75b	0.69	0.25	-1.14	0.42	0.22	-0.69	0.11	0.16	1.24	1.00
tai75c	0.43	0.16	-0.93	0.49	0.25	-0.02	0.11	0.15	2.30	3.00
tai75d	0.43	0.30	0.55	0.49	0.29	-0.23	0.09	0.13	1.55	0.25
tai100a	0.39	0.30	0.61	0.57	0.25	-0.27	0.11	0.18	2.04	1.00
tai100b	0.46	0.29	0.46	0.46	0.22	0.56	0.12	0.16	1.64	1.00
tai100c	0.63	0.27	-0.91	0.50	0.20	-1.10	0.10	0.14	1.34	5.00
tai100d	0.44	0.23	-0.39	0.46	0.21	0.06	0.10	0.17	2.37	2.00
tai150a	0.47	0.29	0.17	0.55	0.31	-0.36	0.09	0.14	2.32	1.67
tai150b	0.61	0.23	0.13	0.52	0.24	0.81	0.09	0.14	2.30	6.00
tai150c	0.47	0.23	0.46	0.61	0.14	-0.06	0.09	0.13	1.80	6.00
tai150d	0.47	0.34	-0.05	0.67	0.23	-0.88	0.12	0.16	1.41	0.14

■ Volume features:

- μ_s - mean volume,
- sd_s - standard deviation of volume,
- $skew_s$ - skewness of volume,
- m_v - minimum number of vehicles necessary to load all the requests.

In order to make the features comparable between different benchmarks, the spatial locations have been mapped to the $[0, 1] \times [0, 1]$ plain, requests volume has been divided by vehicles capacity and k_{gap} has been combined with the m_v in the following way:

$$nc = \left| 1 - \frac{m_v}{k_{gap}} \right|$$

The larger values of nc suggest that the requests might not be easily divided among the vehicles.

The results of computing those features on the Kilby’s and Montemanni’s benchmark set for the *a priori* available requests are presented in Table 1.

4.2 Prediction Model

In order to choose a proper optimization algorithm, the authors propose a simple linear regression model.

To train such a model, for predicting the proper optimization algorithm for a given benchmark, the statistics presented in Table 1 has been selected as an input variables and a ratio between the average MEMSO and the average 2MPSO result has been chosen as an output variable. Such approach is justified by the fact that it is more important to choose a proper algorithm when the difference between different algorithms performance is significant.

Fitted linear model is used in a following way. The subset of the statistics to be computed, described in Section 4.1, is identified with the usage of a variable selection method (cf. Fig. 3). When a new DVRP instance is considered, that subset of statistics, forming the input variables for the model, is computed over the requests of that instance. If the model predicts the relative result of MEMSO to 2MPSO to be less than 1, MEMSO is chosen as the optimization algorithm, otherwise 2MPSO is chosen (see Fig. 2).

5. Results

To test the proposed approach a leave-one-out cross-validation experiment with a linear model predicting the relative performance of the MEMSO to 2MPSO has been performed. In each fold a linear model has been built on information from initial sets of requests from 20 out of 21 benchmark problems. The relative algorithm performance has been computed as a ratio of the average results obtained by the MEMSO and 2MPSO algorithms. Such approach simulated a performance of hyper-heuristic optimizing and training the model on some number of subsequently computed DVRP tasks. The average results were computed over 30 runs of MEMSO per benchmark and 20 runs of 2MPSO per benchmark. The order of the total fitness function evaluations budget for each of the algorithms run has been equal to 10^6 . The details about parameter setting for MEMSO and 2MPSO experiments can be found in [13] and [17], respectively.

Additionally, in the model training phase some of the input variables have been removed from the model in a step-wise mode, with the usage of Akaike Information Criterion, in order to create a more general predictor. As an example, results of applying such procedure, to the model trained on all 21 of the benchmarks, are presented as a listing of R output in Fig. 3. It can be observed (from the corresponding p -values) that

Table 2: Results of predicting the optimization algorithm from a leave-one-out cross-validation experiment. The table presents the minimum and the average values achieved by MEMSO and 2MPSO algorithms, marking the significantly better average results with a gray background. The Hyper-heuristic column presents which algorithm has been chosen by a linear model, whether the choice has been appropriate and how much has been gained (or lost) with that choice of algorithm. For the benchmark instances with significant difference between average results of MEMSO and 2MPSO the gain has been marked with a gray background and a loss with a light-gray background.

Name	2MPSO		MEMSO		Hyper-heuristic		
	min	avg	min	avg	Chosen	T/F	Gain
c50	583.09	618.59	577.60	592.95	MEMSO	T	4.15%
c75	904.83	946.85	928.53	962.54	2MPSO	T	1.63%
c100	926.10	966.27	949.83	968.92	MEMSO	F	-0.27%
c100b	830.58	875.47	864.19	878.81	MEMSO	F	-0.38%
c120	1061.84	1176.38	1164.63	1284.62	2MPSO	T	8.43%
c150	1132.12	1208.60	1274.33	1327.24	2MPSO	T	8.94%
c199	1371.61	1458.01	1600.57	1649.17	2MPSO	T	11.59%
f71	302.50	319.01	283.43	294.85	2MPSO	F	-7.57%
fl34	11944.86	12416.65	14814.10	16083.82	2MPSO	T	22.80%
tai75a	1721.81	1846.03	1785.11	1837.00	2MPSO	F	-0.49%
tai75b	1418.82	1451.92	1398.68	1425.80	MEMSO	T	1.80%
tai75c	1456.90	1560.68	1490.32	1532.45	MEMSO	T	1.81%
tai75d	1445.58	1481.25	1342.26	1448.19	MEMSO	T	2.23%
tai100a	2211.30	2327.20	2170.54	2213.75	MEMSO	T	4.87%
tai100b	2052.54	2131.91	2093.54	2190.01	2MPSO	T	2.65%
tai100c	1465.06	1519.44	1491.13	1553.55	2MPSO	T	2.20%
tai100d	1722.16	1808.67	1732.38	1895.42	2MPSO	T	4.58%
tai150a	3367.55	3537.81	3253.77	3369.48	2MPSO	F	-4.76%
tai150b	2911.22	3033.83	2865.17	2959.15	2MPSO	F	-2.46%
tai150c	2510.51	2579.72	2510.13	2644.69	2MPSO	T	2.46%
tai150d	2893.54	2992.53	2872.80	3006.88	MEMSO	F	-0.48%

both the spatial and the volume related features have been selected as informative ones.

The results from the cross-validation experiment are given in Table 2. The correct algorithm (the one with a better average result) has been chosen in 14 out of 21 cases (all of them with statistically significant difference between average algorithms results, verified by a t -test). Wrong predictions, that have been made for the other 7 cases, have resulted in a significant loss of results quality only for 3 of them (f71, tai150a, and tai150b). Therefore, the linear model achieved 82% accuracy for the subset of 17 benchmarks with significant differences in the algorithms average results (with 67% accuracy over the whole set of benchmarks).

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Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.42899    0.15676   9.116 9.64e-07 ***
sd_x         1.23119    0.26551   4.637 0.000573 ***
mu_y        -1.11031    0.16881  -6.577 2.62e-05 ***
sd_y        -1.56756    0.28836  -5.436 0.000151 ***
skew_y      -0.04576    0.02481  -1.844 0.089977 .
mu_s         2.72795    1.25435   2.175 0.050362 .
sd_s        -3.39358    0.71918  -4.719 0.000498 ***
skew_s       0.21507    0.04592   4.683 0.000529 ***
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Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.'
Residual standard error:
                    0.03915 on 12 degrees of freedom
Multiple R-squared:  0.86, Adjusted R-squared:  0.7764
F-statistic: 10.43 on 7 and 12 DF, p-value: 0.000286

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Figure 3: Model trained on the full data set, with the variable selection using Akaike Information Criterion, presented as an R output.

6. Conclusion

Choosing an optimization algorithm for the DVRP on the basis of the characteristics of initial requests sets leads to the improvement of the results. Choosing between MEMSO and 2MPSO algorithms resulted in a 0.6% improvement in comparison with the 2MPSO average performance (with a maximum of 1.5% improvement if all the predictions were accurate) and 2.8% in comparison with the MEMSO performance. The best performance has been improved by 0.2% on average in comparison with 2MPSO and by 2.8% in comparison with MEMSO.

The obtained results suggest that the proper benchmark features has been selected and choosing an optimization algorithm for a given benchmark problem is possible and may lead to results improvement. Both the spatial and the volume related features have been marked as significant in predicting relative performance.

The future work should include an algorithm choice possibility during the optimization and not only at the beginning of the working day. It might be also beneficial to search for another set of characteristics allowing for a proper choice of the algorithm, in order to try eliminating the DVRP tasks for which the significantly worse algorithm has been selected.

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